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THE NONLINEAR EVOLUTION OF RESISTIVE INTERCHANGE MODES IN REVERSED FIELD PINCHES

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THE NONLINEAR EVOLUTION OF RESISTIVE INTERCHANGE MODES IN REVERSED FIELD PINCHES* — D.D. Schnack and J. Killeen, National MFE Computer Center, Lawrence Livermore Laboratory, Livermore, Ca. 94550

1. INTRODUCTION

The linear stability of cylindrical Reversed Field Pinch configurations is well known [1]. It has been found that while stability against ideal MHD modes can be achieved, such equilibria may be susceptable to resistive instabilities [2]. Recently, analytic equilibria have been discovered which are not only stable against tearing modes at zero-β, but which also satisfy the Suydam criterion for values of central β up to 18% [3]. This is achieved by judiciously expanding the pitch $\mu(r)=rB_z/B_\theta$ as a power series in r which approximates the well known Bessel function model (BFM) near the axis. Unfortunately, these equilibria have been found to be unstable to slow resistive interchange modes [4]. These modes are driven by the local pressure gradient at the singular surface and have a growth rate $\omega \sim t_R^{-1/3} t_H^{-2/3}$, as compared with tearing modes which are driven by the gross configuration of the field away from the singular surface, and which grow as $t_R^{-2/5}\, t_H^{-3/5}$. If these modes are dangerous nonlinearly, they may limit the attainable values of β.

In this paper we present the results of the application of a two dimensional resistive MHD computer code [5] to the nonlinear evolution of resistive interchange modes in tearing-mode-stable RFP equilibria [3]. We find that the m=1 mode is insignificant when the singular surface is outside the field reversal point, and is more active nonlinearly but still fairly localized when the singular surface lies in the inner regions of the plasma. The m=0 mode, which is not present in tokamaks, is found to lead to highly distorted flux surfaces and interchange vortices of large radial extent when β is near the Suydam marginal point. However, if the initial β is sufficiently small, this mode remains localized allowing significant Ohmic heating of the pinch to occur.

2. EQUILIBRIUM AND COMPUTATIONAL MODEL

In cylindrical geometry, the condition for magnetostatic equilibrium can be written in terms of the pitch function $\mu(r)=rB_z/B_\theta$ as

$$\frac{dB_\theta}{dr} = B_\theta \frac{\mu^2/r - \mu\mu' + r\mu'^2 C_1}{\mu^2 + r^2}$$

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where $C_1 = -p'(\mu/\mu')^2/2rB_z^2$, and primes denote differentiation with respect to r. Specific equilibria are found by giving the constant C_1 and the function $\mu(r)$. Note that for $C_1 < 1/8$ the configuration is stable to Suydam modes. Stability against non-localized ideal MHD modes is obtained by either requiring that the pitch length of the magnetic field lines $2\pi\mu$ be greater than the axial wave number of the perturbation, and $\gamma \equiv \mu\mu'/2 > 0$ on axis (as is the case for tokamaks), or that $\mu(r)$ be a monotonically decreasing function of radius which changes sign inside the plasma, and which satisfies $\gamma < -4/9$ (as is the case for the RFP)[1].

In this work we take [3]

$$\mu(r) = 2\,(1 - r^2/8 - r^4/\lambda - r^6/\delta - r^8/\varepsilon - \ldots)$$

where λ, δ, and ε are constants. The motivation for this choice is to obtain a function $\sigma(r) = \underline{J}\cdot\underline{B}/B^2$ which is small in the outer regions of the plasma and approximately constant near the axis. This assures that the tearing mode driving term σ' is near zero in the central regions of the pinch. With $C_1 = 0$, it has been found [3] that the choice of $\lambda = 400$, $\delta, \varepsilon \to \infty$ is optimum in that it yields the most extensive stable region in (k_z, R_W) space. Choosing $C_1 = .1$ results in an equilibrium which is both tearing mode and Suydam stable with a value of central $\beta \approx 18\%$. However, it is now susceptable to pressure driven resistive interchanges, or g-modes.

We study the non-linear evolution of these modes by posing an initial value problem in which the initial conditions consist of the equilibrium quantities described above perturbed by an unstable eigenmode obtained from a numerical solution of the linearized resistive MHD equations [2]. The system is then advanced in time by numerically solving the full set of nonlinear resistive MHD equations in two space dimensions by the method of finite differences [5]. For cases involving axial symmetry (e.g., the m=0 mode), we solve the problem in the (r,z) plane. When m=1, we reduce the dimensionality of the problem from three to two by transforming to the helical coordinate system $(r, \phi = m\theta + k_z z)$.

3. RESULTS

For tearing instabilities at zero β it has been established that the most nonlinearly active mode in both tokamaks and Reversed Field Pinches is characterized by azimuthal mode number m=1 [6,7]. In that case the magnetic island is observed to dominate the central core of the plasma resulting in a final state which is almost asixymmetric. In contrast, we find here that for the resistive g-mode at $\beta = .18$, i.e., near the marginal stability point, the magnetic island for m=1 remains fairly localized near the singular surface. In fact, we find that when the singular surface is outside the field null we are able to observe only the slightest nonlinear growth before complete saturation occurs. However, when the singular surface is inside the field reversal point, the mode is found to have more robust nonlinear behavior. The island in the saturated state for this case is shown in figure 1. Note that, while the island width has become a significant fraction of the minor radius, it is still fairly localized near the singular surface. This difference in nonlinear behavior between m=1 tearing and resistive interchange modes may be due to the fact that the driving force for the g-mode is centered within the resistive layer at the singular surface, whereas the free energy for tearing modes comes from the magnetic field configuration away from the resistive layer.

The case m=0 requires a field null for its occurance and is therefore unique to the RFP. The m=0 tearing mode at zero β has been found to have relatively benign nonlinear behavior [7]. In contrast, the m=0 resistive g-mode, when it evolves near the marginal point β =.18, is found to result in large magnetic islands, highly distorted flux surfaces, and interchange vortices of large radial extent. The flux surfaces and flow field near saturation for this case are shown in figures 2 and 3. Note that the vortex motion tends to become concentrated near the reconnection point, which may indicate a cascading of energy to shorter wavelengths resulting in turbulent flow. This behavior may be due to spatially localized coupling to ideal interchange motion. In any case, such a configuration would seem to be highly conducive to enhanced radial transport. Recently, this behavior has been confirmed with an ion PIC code [8].

The above results indicate that the resistive g-mode may place a restriction on the value of β attainable through Ohmic heating. By requiring that the characteristic Ohmic heating time be less than the e-folding time for a resistive g-mode, one finds that $\beta \lesssim (S/4)^{-2/5}$, where $S=t_R/t_H$ is the magnetic Reynold's number [9]. For $S=10^3$, which characterizes the cases presented above, this requires $\beta \lesssim 0.1$, while for an initially hotter plasma, $S=10^5$, one finds $\beta \lesssim .02$. We investigate this behavior by studying the nonlinear evolution of the m=0 resistive g-mode for the case $S=10^3$, $\beta=.078$, which is less than the limit discussed above. Additionally, we require that the total axial current remain constant. In figure 4 we plot the evolution of $T_0(t)/T_0(0)$, the ratio of the temperature on axis to the initial temperature on axis, and $(W(t)-W(0))/W(0)$, the percentage change in magnetic island width. It can be seen that for this value of β the island saturates quickly allowing significant Ohmic heating to take place. It would thus appear crucial that the initial pinch plasma be formed in a low β state.

4. ACKNOWLEDGMENT

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FIGURE CAPTIONS

Fig. 1 Saturated magnetic island for $m=1$, $k_z=-1$, $\beta=.18$, $S=10^3$.

Fig. 2 Highly nonlinear phase of the magnetic island for the case $m=0$, $k_z=.4$, $\beta=.18$, $S=10^3$.

Fig. 3 Velocity field corresponding to figure 2.

Fig. 4 Relative temperature on axis and percentage change in island width for the case $m=0$, $k_z=1$, $\beta=.078$, $S=10^3$.

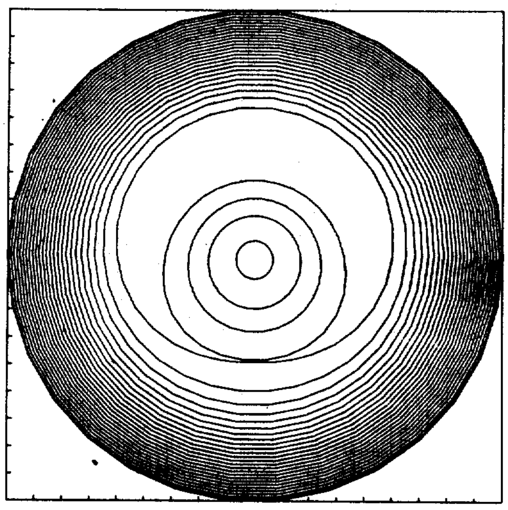

Fig. 1

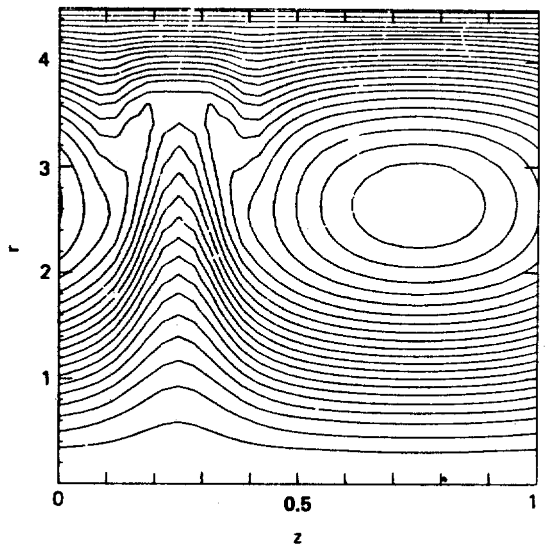

Fig. 2

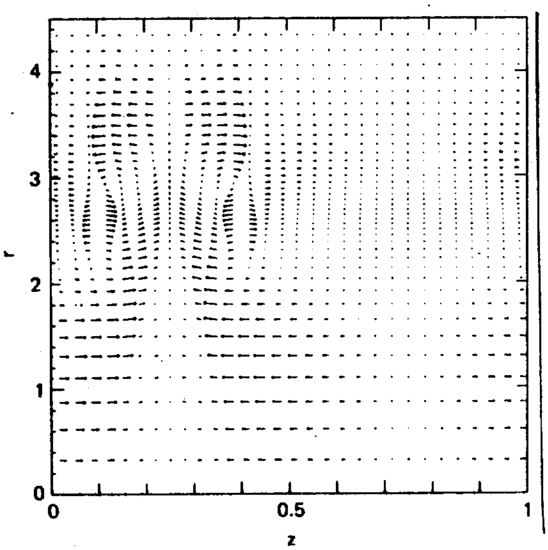

Fig. 3

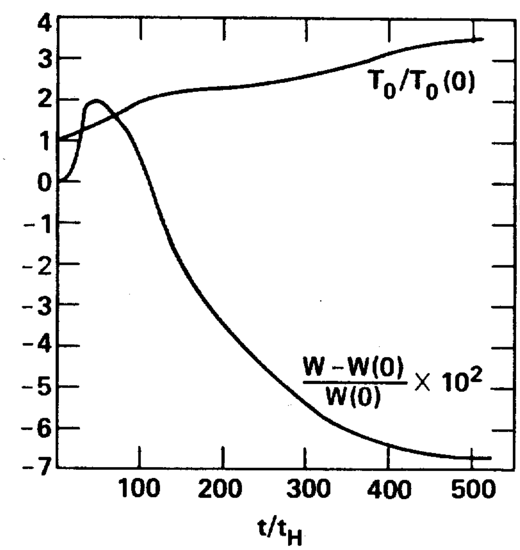

Fig. 4